# Chapter 13 – Comparing Three or More Means

## OUTLINE

1. Comparing Three or More Means: One-Way Analysis of Variance
2. Post-Hoc Tests on One-Way Analysis of Variance

## Putting It Together

Do you remember the progression for comparing proportions? Chapters 9 and 10 discussed inference for a single proportion, Chapter 11 discussed inference for two proportions, and Chapter 12 presented a discussion of inference for three or more proportions (homogeneity of proportions).

We have this same progression of topics for inference on means. In Chapters 9 and 10, we discussed inferential techniques for a single population mean. In Chapter 11, we discussed inferential techniques for comparing two means. In this chapter, we learn inferential techniques for comparing three or more means.

Just as we used a different distribution to compare multiple proportions (the chi-square distribution), we use a different distribution for comparing three or more means. Although the choice of distribution initially may seem strange, once the logic of the procedure is understood, the choice of distribution makes sense.

## Section 13.1 Comparing Three or More Means: One-Way Analysis of Variance

### Objectives

1. Verify the Requirements to Perform a One-Way ANOVA
2. Test a Hypothesis Regarding Three or More Means Using One-Way ANOVA

Introduction, Page 2

1. What is Analysis of Variance (ANOVA) used to test?
2. What are the null and alternative hypotheses for testing a hypothesis regarding three population means?

Introduction, Page 3

1. Sketch an example of what the distribution of the populations might look like if the alternative hypothesis (at least one population mean is different from the others) is true.

Introduction, Page 4

1. Why do we use ANOVA to test the equality of three or more means rather than comparing them two at a time using the techniques used in Section 11.3??

Introduction, Page 5

**Notes on Analysis of Variance**

Sir Ronald A. Fisher (1890–1962) introduced the method called analysis of variance (ANOVA). This term may seem odd because we are conducting a test on means, not variances. However, the name refers to the approach we are using, which will involve a comparison of two estimates of the same population variance. The justification for the name will become clear as we develop the test statistic.

The procedure used in this section is called one-way analysis of variance because only one factor distinguishes the various populations in the study.

We use a one-way analysis of variance when analyzing data from a completely randomized design with three or more levels of treatment. In this design, the subjects must be similar in all characteristics except the level of the treatment.

#### Objective 1: Verify the Requirements to Perform a One-Way ANOVA

Objective 1, Page 1

1. State the four requirements to perform a one-way ANOVA test.

Objective 1, Page 2

1. Explain how to verify the requirement of normality for a one-way ANOVA test.
2. State the rule of thumb for verifying the requirement of equal population variances for a one-way ANOVA test.

Objective 1, Page 4

**Example 1 *Verifying the Requirements of One-Way ANOVA***

Prosthodentists specialize in the restoration of oral function, including the use of dental implants, veneers, dentures, and crowns. Because repair of chipped veneer is less costly and time-consuming than complete restoration, a researcher wanted to compare the shear bond strength of different kits for repairs of chipped porcelain veneer in fixed prosthodontics. He randomly divided 20 porcelain specimens into four treatment groups. Group 1 specimens used the Cojet system, group 2 used the Silistor system, group 3 used the Cimara system, and group 4 specimens used the Ceramic Repair system. At the conclusion of the study, shear bond strength (in megapascals, MPa) was measured according to ISO 10477. The data in Table 1 are based on the results of the study. Verify that the requirements to perform one-way ANOVA are satisfied.

**Table 1**

| Cojet | Silistor | Cimara | Ceramic Repair |
| --- | --- | --- | --- |
| 15.4 | 17.2 | 5.5 | 11.0 |
| 12.9 | 14.3 | 7.7 | 12.4 |
| 17.2 | 17.6 | 12.2 | 13.5 |
| 16.6 | 21.6 | 11.4 | 8.9 |
| 19.3 | 17.5 | 16.4 | 8.1 |

Data from P. Schmage et al. “Shear Bond Strengths of Five Intraoral Porcelain Repair Systems,” Journal of Adhesion Science & Technology 21 (5–6):409–422, 2007

#### Objective 2: Test a Hypothesis Regarding Three or More Means Using One-Way ANOVA

Objective 2, Page 1

 *Answer the following after watching the video.*

1. Explain what is meant by the terms between-sample variability and within-sample variability.
2. What evidence will suggest that the samples come from populations with different means?

Objective 2, Page 3

 *Watch the video to learn how to compute the F-test statistic.*

Objective 2, Page 4

1. State the formulas for the mean square due to error (MSE), the mean square due to treatment (MST), and the test statistic for a one-way ANOVA test .

Objective 2, Page 5

1. List the six steps for computing the F-test statistic by hand.

Objective 2, Page 6

**Example 2 *Computing the F-Test Statistic by Hand***

Compute the F-test statistic for the data shown

|  |  |  |
| --- | --- | --- |
| 4 | 7 | 10 |
| 5 | 8 | 10 |
| 6 | 9 | 11 |
| 6 | 7 | 11 |
| 4 | 9 | 13 |

Objective 2, Page 8

The data from Example 2 was from Table (a) in the video that presented a conceptual understanding of one-way ANOVA video, and the *F*-test statistic was 38.57.

For the data in Table (b) in the conceptual understanding of one-way ANOVA video, MST= 45, MSE = 24.1667, and the *F*-test statistic is 1.86.

The small *F*-test statistic for the data in Table (b) is evidence that the sample means for each treatment do not differ.

Objective 2, Page 9

**Note:** The computations that lead to the *F*-test statistic are presented in Table 2, which is called an ANOVA table.

**Table 2**

| **Source of Variation** | **Sum of Squares** | **Degrees of Freedom** | **Mean Squares** | ***F*-Test Statistic** |
| --- | --- | --- | --- | --- |
| Treatment | *SST* |  |  |  |
| Error | *SSE* |  |  |  |
| Total | *SST + SSE* |  |  |  |

Objective 2, Page 11

1. State the decision rule for a one-way ANOVA test.

Objective 2, Page 12

**Example 3 *Performing One-Way ANOVA Using Technology***

Prosthodentists specialize in the restoration of oral function, including the use of dental implants, veneers, dentures, and crowns. Since repairing chipped veneer is less costly and time consuming than complete restoration, a researcher wanted to compare the shear bond strength of different kits for repairs of chipped porcelain veneer in fixed prosthodontics. He randomly divided 20 porcelain specimens into four treatment groups. Group 1 specimens used the Cojet system, group 2 used the Silistor system, group 3 used the Cimara system, and group 4 specimens used the Ceramic Repair system. At the conclusion of the study, shear bond strength (in megapascals, MPa) was measured according to ISO 10477. The data in Table 3 are based on the results of the study. Do the data suggest that there is a difference in the mean shear bond strength among the four treatment groups at the  level of significance?

**Table 3**

| **Cojet** | **Silistor** | **Cimara** | **Ceramic Repair** |
| --- | --- | --- | --- |
| 15.4 | 17.2 | 5.5 | 11.0 |
| 12.9 | 14.3 | 7.7 | 12.4 |
| 17.2 | 17.6 | 12.2 | 13.5 |
| 16.6 | 21.6 | 11.4 | 8.9 |
| 19.3 | 17.5 | 16.4 | 8.1 |

Data from P. Schmage et al. “Shear Bond Strengths of Five Intraoral Porcelain Repair Systems,” Journal of Adhesion Science & Technology 21 (5–6):409–422, 2007

Objective 2, Page 13

**Note:** Whenever performing analysis of variance, it is a good idea to present visual evidence that supports the conclusions of the test. Side-by-side boxplots are a great way to help visually reinforce the results of the ANOVA procedure.

Objective 2, Page 15

**Note:** When we reject the null hypothesis of equal population means, as in Example 3, we know that at least one population mean differs from the others. However, we do not know which means differ.

Side-by-side boxplots can give us some idea, but we can more formally answer this question using Tukey's test, which will be discussed in the next section.

Objective 2, Page 16

**Note:** Another way to verify the normality requirement in a one-way ANOVA test is through a residual plot. The residual of each value is the difference between the value and its sample’s mean. Once the residual has been calculated for each value in each sample, create a normal probability plot for the residuals. If the residuals are found to be normally distributed, the normality requirement is satisfied.

Objective 2, Page 17

**Example 4** *Verifying the Normality Requirement by Analyzing Residuals*

Verify the normality requirement for the data analyzed in Example 3.

**Table 3**

| **Cojet** | **Silistor** | **Cimara** | **Ceramic Repair** |
| --- | --- | --- | --- |
| 15.4 | 17.2 | 5.5 | 11.0 |
| 12.9 | 14.3 | 7.7 | 12.4 |
| 17.2 | 17.6 | 12.2 | 13.5 |
| 16.6 | 21.6 | 11.4 | 8.9 |
| 19.3 | 17.5 | 16.4 | 8.1 |

Data from P. Schmage et al. “Shear Bond Strengths of Five Intraoral Porcelain Repair Systems,” Journal of Adhesion Science & Technology 21 (5–6):409–422, 2007

## Section 13.2 Post-Hoc Tests on One-Way Analysis of Variance

### Objective

1. Perform Tukey’s Test

Introduction, Page 1

Suppose the results of a one-way ANOVA show that at least one population mean is different from the others. To determine which means differ significantly, we make additional comparisons between means using procedures called multiple comparison methods.

#### Objective 1: Perform Tukey’s Test

Objective 1, Page 1

1. What is compared in the Tukey test? What is the goal of the test?
2. State the formula for the standard error when using Tukey’s test.

Objective 1, Page 2

1. State the test statistic for Tukey’s test when testing  versus .

Objective 1, Page 3

**Note:** The critical value for Tukey's test using a familywise error rate α is given by



where

*v* is the degrees of freedom due to error, which is the total number of subjects sampled minus the number of means being compared, or 

*k* is the total number of means being compared

We can determine the critical value from the Studentized range distribution by referring to Table X.

Objective 1, Page 4

**Example 1 *Finding Critical Values from the Studentized Range Distribution***

Find the critical value from the Studentized range distribution with *v* = 7 degrees of freedom and *k* = 3 degrees of freedom, with a familywise error rate 

Objective 1, Page 6

1. After rejecting the null hypothesis , what are the six steps when performing Tukey’s test?

Step 1

Step 2

Step 3

Step 4

Step 5

Step 6

Objective 1, Page 7

**Example 2 *Performing Tukey’s Test by Hand***

In Example 3 from Section 13.1, we rejected the null hypothesis **. Use Tukey's test to determine which pairwise means differ using a familywise error rate of α= 0.05.

Objective 1, Page 8

**Example 3 *Performing Tukey's Test Using Technology***

In Example 3 from Section 13.1, we rejected the null hypothesis. Use StatCrunch to conduct Tukey's test to determine which pairwise means differ using a familywise error rate of .

Objective 1, Page 10

Sometimes the results of Tukey’s test are ambiguous. Suppose the null hypothesis  is rejected and the results of Tukey’s test indicate the following:



We can conclude from this result that , but we cannot tell how  is related to , , or . A solution to this problem is to increase the sample size so that the test is more powerful.

It can also happen that the one-way ANOVA rejects  but the Tukey test does not detect any pairwise differences. The result occurs because one-way ANOVA is more powerful than Tukey’s test. Again, the solution is to increase the sample size.